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Resale Price Maintenance in a Successive Monopoly Model*

Markus Dertwinkel-Kalt† Christian Wey‡

February 2023

Abstract

We present a model to explain why a manufacturer may impose a minimum resale price (min RPM) in a successive monopoly setting. Our argument relies on the retailer having non-contractible choice variables, which could represent the price of a substitute good and/or the effort the retailer exerts for service provision or advertising. Our explanation for a min RPM is empirically distinguishable from alternative justifications for a min RPM that rely, for instance, on retailer competition and service free riding among retailers. Whether a min RPM benefits or harms consumers depends on—as we show—why a min RPM is implemented: if the goal is to soften competition with the substitute product, it tends to harm consumers, and if the goal is to secure service provision, it tends to benefit consumers.

JEL-Classification: L12, L41, D42, K21.

Keywords: Resale Price Maintenance, Vertical Restraints, Cost Pass-Through, Retailing.

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1 Introduction

1.1 Motivation

Manufacturers often seek to restrain retailers' flexibility in setting retail prices by using resale price maintenance (RPM). An RPM may require the retailer not to lower the price below a certain *minimum* price (in short: min RPM). Conversely, it may specify a certain *maximum* price (in short: max RPM) that the retailer's price must not exceed. As a max RPM straightforwardly helps to overcome the double-markup inefficiency in vertical relations, its treatment in antitrust regulations has been much less restrictive and controversial than the one of min RPM (which includes a price-fixing RPM). However, RPM commonly comes in the form of *minimum* retail prices (see Ippolito, 1991, 2010).

The Supreme Court in *Leegin Creative Leather Products, Inc. v. PSKS, Inc., 551 US 877* (2007) removed the per-se ban on min RPM and replaced it with a rule-of-reason approach in the US, but the legal status of min RPM "is still far from clear today" (Lafontaine and Slade, 2014). Nevertheless, according to the most recent estimates, more than $300 billion in sales alone in the US are affected annually by RPM agreements (Gundlach and Krotz, 2020). While min RPM is considered a hardcore restraint of competition in the European Union and therefore illegal, there is nevertheless empirical support for the usage of it also in the European Union (Bonnet and Dubois, 2010). As a min RPM protects the retailer's margin and keeps demand relatively low, how can it be explained?

Existing explanations build on the effect that a min RPM softens intrabrand competition among independent retailers and therefore tends to be welfare-decreasing. On the other hand, softening intrabrand-competition can be welfare-enhancing if a min RPM counters retailer service free riding and thereby protects the provision of retailer services (Telser, 1960; Mathewson and Winter, 1984). In practice, however, min RPM is also applied to a range of products for which service free riding is not plausible (see Pitofski, 1983; Ippolito, 1991; MacKay and Smith, 2017), and where, more generally, intrabrand competition does not seem to be important; for instance, when an RPM is combined with a territorial exclusivity clause (see Boyd, 1993, Table

II, p. 761).[1] Nevertheless, many more cases could be expected if min RPM would not have been illegal for a long time and if its legal status was not "far from clear today" (Lafontain and Slade, 2014). So, why is the implementation of min RPM also desirable when intrabrand competition is no major concern? Put differently, are there other explanations for min RPM that do not focus on softening intrabrand competition?

In this paper, we establish such a reason for RPM in a successive monopoly model (Spengler, 1950), which we augment by considering additional decision variables the retailer has at hand and which cannot be (contractually) controlled by the manufacturer. We consider two cases, first separately and later in combination, namely, the case of multiproduct retailing (where the retailer sets the price of a substitute product) and the case of demand-enhancing selling services. Our main assumption is that a fixed upfront payment is not feasible (or sufficiently constrained), so a linear wholesale price is the only instrument the supplier has to extract rents from the retailer. It follows that the retailer's additional decisions create a vertical externality, which necessarily affects the manufacturer's profit.

When the manufacturer sets the retail price (and hence, controls the retail margin), the manufacturer faces a tradeoff between reducing the double-markup problem and incentivizing the retailer to internalize the vertical externality that comes from his other decision variable(s). We show that the resolution of this tradeoff depends on the *induced* demand of the retailer (which results from the retailer's optimal choice of his additional decision variable(s)), such that an RPM is always used to increase the sales quantity of the manufacturer's product. It then follows that a min (max) RPM is chosen whenever the induced demand of the manufacturer's good is upward (downward) sloping in its own retail price. Intuitively, a min RPM (max RPM) is optimal

[1] Territorial supply constraints can prevent intrabrand competition by inhibiting other retailers from selling the same products in a certain territory. Building on an FTC study of 1988 (Ippolito, 1988), Boyd (1993) lists a number of cases where min RPM was used when territorial supply constraints were in place. Ippolito (1988) presents in Table A1 an extensive list of "Cases With a Vertical Price-Fixing Charge, 1976-1982". For each considered case, the table provides information about the "RPM type" and "Other Vertical Charges" as, e.g., exclusive territories. Presumably, the exclusive territories provision helped to establish a monopoly for the distributor—at least by restricting intrabrand competition. Proceeding this way, Boyd (1993, Table II, p. 761) shows that in 27 cases (out of a total of 113 considered cases) RPM and exclusive territories were both used together.

whenever the vertical externality associated with the retailer's additional decision variable is relatively more (less) important for the manufacturer's sales quantity than the double-markup problem.

For the case of the multiproduct retailer, the profitability of a min RPM follows directly from observing that the retailer's *induced* demand for the manufacturer's good can be *increasing* in its retail price for a standard "downward sloping" demand system. Put another way, with a min RPM, the manufacturer can induce an increase of *all* retail prices, which drives relatively high-value consumers back to the manufacturer's brand and thereby increases its sales volume. This explanation of a min RPM is related to the "exclusivity/prestige" argument (or "image theory") in favor of a min RPM, which postulates that consumer demand for a brand increases in its price (see Orbach, 2010; Inderst, 2019). Interestingly, our argument for a min RPM also relies on an "upward sloping demand" mechanism; but it is now the retailer's *induced* demand which may increase in the manufacturer's retail price, while consumer demands are downward sloping as usual. Moreover, while a min RPM can be socially desirable when "prestige" matters, it tends to harm consumers in our setting as it raises all retail prices; this is always the case in the multiproduct case when demand is linear and a min RPM is optimal for the manufacturer.

For the case of the effort-providing retailer, we show that a min RPM can incentivize the retailer to provide more effort, even when intrabrand competition and service free riding by other retailers is no concern. Here, a min RPM tends to benefit consumers because the benefit from increased service provision outweighs the harm of a higher retail price. However, we also show that the induced service provision level can be excessive from a social welfare perspective.[2] In a generalized setting, where the retailer decides both about another product's price and product-specific selling services for the manufacturer's product, a min RPM unfolds an anti-competitive effect (concerning the other good's price) and a pro-competitive effect (concerning selling services), so that consumers can be worse off than in a regime that bans RPM. In contrast, a max RPM allows overcoming the double-markup problem, which induces the retailer to reduce not only the substitute's price but also the selling services. Because of the tradeoffs between

[2]Likewise, a max RPM can be used in a socially inefficient way if it excessively reduces the retailer's service effort.

overcoming the double-marginalization problem and incentivizing the retailer, both max and min RPM can lower consumer and social welfare.

Notably, we explain the usage of a min and a max RPM via the slope of induced demand, which is not observable. We show, however, that the induced demand is upward (downward) sloping if and only if the cost pass-through with regard to the manufacturer's product is negative (positive) in a regulatory environment where RPM is not feasible. Thus, we have linked the unobservable sign of the slope of induced demand to the better measurable cost pass-through. Given a regulatory environment that effectively prohibits RPM, the counterfactual RPM scenario can be inferred relatively easily from the cost pass-through behavior of the retailer.

1.2 Related Literature

Our explanation for the usage of a min RPM is complementary to other explanations for the use of RPM clauses in vertical relations. By large, the relevant literature can be divided into two strands, one highlighting their pro-competitive effects and the other one providing theories of harm that delineate their anti-competitive nature. According to the former strand, a min RPM can be desirable in settings with intrabrand competition as it could counter retailer service free riding and thereby protect the provision of retailer services (see discussion above),[3] and it could help to avoid destructive retailer competition (Deneckere et al., 1997).[4]

The literature that deals with the anti-competitive effects of min RPM has singled out the following anti-competitive mechanisms, which are largely surveyed in Marvel (1994), Rey and Vergé (2008), Elzinga and Mills (2008), and Bennett et al. (2011). Min RPM can weaken intrabrand competition as a facilitating practice for downstream collusion, and it could weaken interbrand competition as a facilitating practice for upstream collusion (Jullien and Rey, 2007;

[3]Relatedly, Marvel and McCafferty (1984) have shown that a manufacturer can benefit from RPM, as retailers with a high reputation (that signals quality to consumers) can be incentivized to sell the product.

[4]Winter (1993) shows that a min RPM contract is also optimal when retailers' sales efforts do not exhibit a public good character, as there otherwise would be excessive price competition. Ippolito and Overstreet (1996) mention the expansion of the distribution network as another pro-competitive effect of an RPM. See also Klein and Murphy (1988) for transaction cost-based arguments in favor of vertical restraints as a means to promote a manufacturer's good at retailers premises.

Hunold and Muthers, 2020) as well as for the exclusion of lower-cost rival firms (Asker and Bar-Isaac, 2014). Innes and Hamilton (2009) analyze a model of multiproduct retail competition and show how an RPM contract (together with a two-part tariff) can be used to appropriate rents from the other retailer's product. Industrywide min RPM can also serve as a commitment device to protect upstream monopoly rents, which is an issue under secret contracting (see Hart and Tirole 1990; O'Brien and Shaffer, 1992; Rey and Vergé, 2004; Gabrielsen and Johansen, 2017). Moreover, a min RPM can benefit downstream firms by making it harder for entrants to steal business away by undercutting (Shaffer, 1991). Min RPM can also eliminate all effective competition—at the interbrand level as well as at the intrabrand level—through networks of interlocking RPM agreements in a setting with two manufacturers and common retailers (Dobson and Waterson, 2007; Rey and Vergé, 2010). In this setup, Hunold and Muthers (2017) also challenge the service argument as an efficiency defense for a min RPM by showing that if manufacturer market power is asymmetric, a min RPM may distort the allocation of services toward the high-priced products of the manufacturer with more market power.[5] Our explanation for a min RPM can be empirically distinguished from all the preceding explanations as it (i) does not rely on either competition on the side of the retailers or on retailer service free riding, so prevails absent intrabrand competition, and (ii) does not rely on manufacturers using it as some coordination device by implementing it mutually.

Thus, our contribution is to show that a min RPM can occur in the archetypal bilateral trading model—that is, in a successive monopoly model as proposed by Spengler (1950)—, which we augment with additional decision variables the retailer has at hand (other goods' prices and/or product-specific selling services), which create vertical externalities. Our model is closely related to the literature that deals with successive monopolies and double moral hazard. Here, in particular, Romano (1994) has analyzed a successive monopoly setting, where both the upstream and the downstream firm make non-contractible choices ("quality" and "promotions", respectively) which affect final good demand. Even though the manufacturer is allowed to use a

[5] Other arguments for the anticompetitiveness of min RPM exist for very different setups as they refer to two-sided markets (Gabrielsen et al., 2018a), to setups where shelf space is costly (Gabrielsen et al., 2018b), or settings where retailers can third-degree price discriminate depending on consumers' ability to switch retailers (Chen, 1999).

non-linear contract (e.g., two-part tariff), it is shown that upstream moral hazard causes double marginalization. Romano then obtains a condition (Proposition 3) for the use of min RPM and max RPM, which is quite similar to ours for the case that the retailer's only non-contractible decision variable is "service" (below we show in detail how our result relates to Romano, 1994).

Starting from Romano (1994), our contribution is to explicitly derive the optimality conditions that yield the equilibrium RPM contract (specifying the wholesale price and the minimum/maximum resale price) and to examine the welfare effects of min and max RPM (which is missing in Romano, 1994). Moreover, we relate our RPM result to the cost-pass-through analysis, and we point out similarities and differences between the two cases where the retailer's non-contractible variable is the price of a rival product and where it is the service level. In addition, we examine the case where the retailer has several non-contractible decision variables, namely, prices of substitute goods and service levels. This gives us new results on the welfare trade-offs associated with RPM; for instance, we derive conditions such that min RPM and max RPM reduce consumer and social welfare.

In the case of multiproduct retailing, the substitute good could reflect a retail brand or a private label good. Such private-label substitute products are widespread, as discussed in the growing literature on multiproduct retailing (see, e.g., Moorthy, 2005; Gabrielsen and Sørgard, 2007; Ezrachi and Bernitz, 2009; Innes and Hamilton, 2009). Moreover, we adopt the assumption of a linear wholesale price,[6] an assumption widely used in the vertical relations literature (see Dobson and Waterson, 2007; Inderst and Valletti, 2009; and Gaudin, 2018). Moreover, a linear wholesale price is not necessary for our results to hold: it is straightforward to show that they also emerge in the case of two-part tariffs when the fixed fee is constrained in such a way that the manufacturer also wants to extract a margin through the wholesale price.[7] Fur-

[6]If (unconstrained) two-part tariff contracts are possible, then an RPM contract is never necessary to achieve the vertically integrated solution within a successive monopoly model. The manufacturer could always set the wholesale price equal to marginal cost and extract all (incremental) retailer surplus via the fixed payment.

[7]The upfront payment could be constrained because of limited commitment on the manufacturer's side (Boyd, 1993), a liquidity-constrained retailer, or risk aversion on the retailer's side. For the latter point, see Rey and Tirole (1986), where the fixed payment is constrained because the retailer is risk averse, which leads to a double-markup problem as with a simple linear wholesale price.

thermore, as mentioned above, it is not uncommon that RPM is used when service free riding and inter-retailer competition is mainly absent; for instance, when used together with territorial exclusivity. Yet, in a setting with a monopoly retailer, a manufacturer could implement the vertically integrated solution with a two-part tariff, which makes an RPM unnecessary. That manufacturers nevertheless used illegal min RPM clauses is, therefore, evidence of the absence of fixed upfront payments.

Finally, our analysis contributes to the debate among antitrust economists about the pros and cons of min RPM in the aftermath of the *Leegin* Supreme Court decision; in particular, the controversy between Klein (2009) and Grimes (2010). Both authors agree that intrabrand competition and service free riding are, in many cases, not really applicable. Klein (2009) argues in favor of RPM as an efficient tool to encourage retailers to supply more manufacturer-specific point-of-sale promotional services; notably, "in the absence of free-riding" (see Klein, 2009, p. 437). In contrast, Grimes (2010) provides a series of arguments highlighting anti-competitive RPM effects, one of which questions the merits of the manufacturer's ability to induce a switch of consumers from rival products to its own product (Grimes, 2010, p. 111). In the generalized version of our successive monopoly model (where the retailer decides about rival goods' prices and product-specific selling services), we can combine both arguments in a single framework, which supports a balanced assessment of min RPM contracts: the retail-margin control associated with an RPM unfolds both a pro-competitive effect on product-specific retailer services and an anti-competitive effect on the price-setting of rival products' prices at the retailer's premises.

In the following, Section 2 presents the model setup. Section 3 provides the general analysis of RPM in a successive monopoly model with vertical externalities, where the retailer has a second decision variable. In Section 4, we relate our main result to the cost-pass-through analysis under an RPM ban, and in Section 5, we provide two examples, one highlighting the anti-competitive effect of a min RPM (the multiproduct case) and one in line with a pro-competitive assessment of a min RPM (the service case). Section 6 generalizes our findings towards a successive monopoly structure, where the retailer sets the prices of more than one other good and the selling service levels for a subset of those goods. In that section, we also provide an illustrative example combining the anti-competitive and the pro-competitive effects of a min

RPM in a single successive monopoly framework. Finally, Section 7 concludes.

2 The Successive Monopoly Model

This section presents the general setup of the successive monopoly structure. Let us consider the contracting problem between a manufacturer M ("she") and a retailer R ("he"). M produces a single good, good 1, at marginal costs $c_1 \geq 0$ and sells it via R to final consumers. The retailer also has a second choice variable $x \geq 0$ that the contract with the manufacturer cannot be conditioned on and which creates a vertical externality. Variable x can either represent (i) the price of a second, substitute good that the retailer produces in-house at marginal costs $w_2 \geq 0$ (we call this the "multiproduct case"),[8] or (ii) the effort level of the retailer for sales, advertising or service provision, which increase consumer demand for the manufacturer's product (we call this the "service case"). Consumer demand for good 1, $q_1 = D_1(p_1, x)$, is continuously differentiable, it is decreasing in its own price ($\frac{\partial D_1}{\partial p_1} < 0$) and increasing in the other choice variable ($\frac{\partial D_1}{\partial x} > 0$).[9] We assume that c_1 is sufficiently small so that there is a gain from trade between M and R.

We suppose that the wholesale price is the only instrument the manufacturer has to extract rents from the retailer. On top of the wholesale price, the manufacturer can impose an RPM clause as a vertical restraint on the retailer. The game is, therefore, as follows. In the first stage, M sets the wholesale price w_1 and a retail price ceiling (max RPM) or a retail price floor (min RPM) for good 1 to the buyer firm. In the second stage, the retailer decides whether to procure good 1 under the posted terms and sets both p_1 and x.

Depending on the nature of x, the profit function of the retailer differs: (i) If x represents the

[8] Alternatively, we may assume that good 2 is supplied at a linear wholesale price w_2 under conditions of perfect competition with constant returns to scale, so that its wholesale price is equal to marginal costs; i.e., $w_2 = c_2$ holds.

[9] Our demand setup is closely related to existing RPM models, which consider either retailer selling services (for instance, Mathewson and Winter, 1984, and Winter, 1993) or multiproduct retailing (see Innes and Hamilton, 2009; Rey and Vergé, 2010). Notably, all those works consider duopoly competition in the retail market (and also a two-part tariff contract instead of a linear wholesale price), so that suppression of intrabrand competition is the main source of min RPM.

price of a second good with final consumer demand $q_2 = D_2(p_1, x)$, with $\frac{\partial D_2}{\partial x} < 0$ and $\frac{\partial D_2}{\partial p_1} > 0$, and procurement costs per unit of w_2, then the profit function equals

$$\pi_R = D_1(p_1, x)(p_1 - w_1) + D_2(p_1, x)(x - w_2), \tag{1}$$

while his outside option profit is $\pi_R^0 := \max_{x \geq 0} D_2(p_1 \to \infty, x)(x - w_2)$. (ii) When x represents some kind of sales effort with $C(x)$ as the service-cost function, with $C(0) = 0$ and $\frac{\partial C}{\partial x} > 0$, then R's profit function is given by

$$\pi_R = D_1(p_1, x)(p_1 - w) - C(x). \tag{2}$$

while his outside option profit is $\pi_R^0 = 0$.

We assume in the following that standard second-order conditions hold when the retailer sets both the retail price for good 1 and the value of his additional decision variable x.

Assumption 1 (Second-order conditions). *Standard second-order conditions of the retailer's (unconstrained) problem,* $\max_{p_1, x \geq 0} \pi_R$*, hold for all* w_1 *not prohibitively large; i.e.,* $\frac{\partial^2 \pi_R}{\partial p_1^2} < 0$, $\frac{\partial^2 \pi_R}{\partial x^2} < 0$, *and* $\frac{\partial^2 \pi_R}{\partial p_1^2} \frac{\partial^2 \pi_R}{\partial x^2} - \left(\frac{\partial^2 \pi_R}{\partial p_1 \partial x}\right)\left(\frac{\partial^2 \pi_R}{\partial x \partial p_1}\right) > 0.$

Assumption 1 ensures that the retailer's profit-maximizing decisions about p_1 and x are uniquely determined by the first-order conditions of his maximization problem. Note that the first-order conditions only hold in an interior optimum, where the optimal retailer decisions lead to strictly positive output levels for both products in the multiproduct case and a strictly positive service level in the service case. Throughout this paper, we assume that this holds. After having presented our setup, we solve the game and present our results in the next section.

3 Analysis and Main RPM Result

We solve for the subgame-perfect Nash equilibrium in two steps. In step 1, we solve the game for a price-fixing RPM, so that the manufacturer determines both the retail price p_1 and the wholesale price w_1. Here, we first solve the second stage of the game to obtain the induced demand for good 1 (step 1a). Secondly, we solve the manufacturer's maximization problem for the optimal wholesale and retail price of good 1 (step 1b). In step 2, we show that the same solution can be implemented with the weaker min RPM or max RPM restraint.

Step 1a: Derivation of the induced demand for good 1. In the second stage of the game, the retailer decides whether to procure good 1 under the posted terms. If the retailer wants to procure good 1, then the retailer chooses x optimally given p_1 and w_1. Assumption 1 ensures that we can write the retailer's profit-maximizing level of x as a function $\widehat{x} := x(p_1, w)$. Given the retailer's optimal response in x, that is, $\widehat{x}$, the induced demand for good 1, $\widehat{q}_1$, is also a function of p_1 and $\widehat{x}$; i.e., the induced demand is given by

$$\widehat{q}_1 = D_1(p_1, \widehat{x}). \tag{3}$$

Taking the total derivative of (3) with respect to p_1 yields

$$\frac{d\widehat{q}_1}{dp_1} = \frac{\partial D_1}{\partial p_1} + \frac{\partial D_1}{\partial x} \cdot \frac{d\widehat{x}}{dp_1}, \tag{4}$$

so that the total demand effect of a price change of good 1 is given by the sum of the direct effect on demand (first term on the right-hand side of (4)) and the indirect effect, which works via the retailer's optimal adjustment of x (second term on the right-hand side of (4)). The slope of the induced demand measures the importance of the retailer's additional decision variable x relative to the usual double-markup problem for the manufacturer's sales quantity. If the retailer's additional decision variable x is relatively unimportant, then the direct demand effect dominates so that there is no case for a min RPM, as the manufacturer only wants to overcome the double-markup problem with the help of a max RPM. If, however, the retailer's additional decision variable becomes more important for the manufacturer's sales quantity—which requires a relatively large positive value of the second term on the right-hand side of (4)—, then the manufacturer may want to impose a min RPM to increase the sales quantity of her good.

Applying the implicit function theorem to the first-order condition $\frac{\partial \pi_R}{\partial x} = 0$ gives for both cases (i) and (ii) the optimal adjustment of x in response to a marginal change of p_1; that is,

$$\frac{d\widehat{x}}{dp_1} = -\frac{\frac{\partial^2 \pi_R}{\partial x \partial p_1}}{\frac{\partial^2 \pi_R}{\partial x^2}}, \tag{5}$$

so that (4) can be written as

$$\frac{d\widehat{q}_1}{dp_1} = \frac{\partial D_1}{\partial p_1} - \frac{\partial D_1}{\partial x} \cdot \frac{\frac{\partial^2 \pi_R}{\partial x \partial p_1}}{\frac{\partial^2 \pi_R}{\partial x^2}}. \tag{6}$$

Thus, the sign of (6) depends on the slope of the retailer's reaction function, $\frac{d\widehat{x}}{dp_1}$, which in turn depends on the sign of $\frac{\partial^2 \pi_R}{\partial x \partial p_1}$. As we assumed $\frac{\partial D_1}{\partial x} > 0$ and $\frac{\partial^2 \pi_R}{\partial x^2} < 0$ (Assumption 1), a necessary condition for a positively sloped induced demand is $\frac{\partial^2 \pi_R}{\partial x \partial p_1} > 0$, which ensures that the retailer's reaction function, $\frac{d\widehat{x}}{dp_1}$, has a positive slope.

In case (i) we have

$$\frac{\partial^2 \pi_R}{\partial x \partial p_1} = \frac{\partial D_1}{\partial x} + \frac{\partial D_2}{\partial p_1} + \frac{\partial^2 D_1}{\partial x \partial p_1}(p_1 - w_1) + \frac{\partial^2 D_2}{\partial x \partial p_1}(x - w_2). \tag{7}$$

The first two terms are positive, while the remaining terms are ambiguous. Note, however, if demand functions are linear, the derivative (7) is always strictly positive because the derivatives in the last two terms are then zero.

And in case (ii) we have

$$\frac{\partial^2 \pi_R}{\partial x \partial p_1} = \frac{\partial D_1}{\partial x} + \frac{\partial^2 D_1}{\partial x \partial p_1}(p_1 - w_1) - \frac{\partial^2 C}{\partial x^2}. \tag{8}$$

Here, the first term is positive, while the remaining terms are ambiguous. If, however, the demand function is linear in x and p, the derivative (8) is always strictly positive. In this case $\frac{\partial^2 C}{\partial x^2} < 0$ follows from Assumption 1, while the second term in (8) is then zero.

Without further information about the specific functional form of the demand system, it is impossible to determine the sign of the slope of the induced demand (6). At least, we can conclude that in the case of a linear demand, the indirect effect of a marginal price increase of good 1 goes in the opposite direction to the direct demand effect. This hints at the possibility that the induced demand could be upward or downward sloping depending on the demand's exact parameters (below, we provide examples to show that this is true for the multiproduct and the service case).

To proceed in a parsimonious way, we invoke the assumption that the induced demand, $\widehat{q}_1$, is monotone in p_1 in the relevant range from the manufacturer's perspective; i.e., we face either the increasing demand ("ID") case or the decreasing demand ("DD") case.

Assumption 2 (Monotonicity of the induced demand function). *The induced demand, $\widehat{q}_1$, is either strictly monotonically increasing in p_1 for all $p_1 \geq w_1 \geq c_1$ (i.e., the ID-case with $\frac{d\widehat{q}_1}{dp_1} > 0$ holds according to (6)) or it is strictly monotonically decreasing in p_1 for all $p_1 \geq w_1 \geq c_1$ (i.e., the DD-case with $\frac{d\widehat{q}_1}{dp_1} < 0$ holds according to (6)).*

The slope of the induced demand (4) is important below, so we show how it can be expressed and interpreted in terms of more familiar elasticities (see Romano, 1994, to which we refer more precisely below). Let $\epsilon_{p_1} := -\frac{\partial D_1}{\partial p_1}\frac{p_1}{q_1}$ denote the own-price elasticity of demand, $\epsilon_x := \frac{\partial D_1}{\partial x}\frac{x}{q_1}$ the elasticity of demand with respect to the retailer's other decision variable (either service or price of good 2), and $\eta_x := \frac{\partial x}{\partial p_1}\frac{p_1}{x}$ denote the price elasticity of the other decision variable. We then get $\frac{d\widehat{q}_1}{dp_1} = \frac{D_1}{p_1}\left(-\epsilon_{p_1} + \epsilon_x \cdot \eta_x\right)$; that is, the sign of the slope of the induced demand depends critically on product 1's own price elasticity and the elasticity of demand with respect to the retailer's other decision variable. The latter is given by the cross-price elasticity of demand of good 1 with respect to the other good's price in the multiproduct case and by the service elasticity of demand of good 1 in the service case.

While a change of p_1 affects the induced demand for good 1 both directly and indirectly (see (6)), a marginal change of w_1 can only affect the demand for good 1, $\widehat{q}_1$, indirectly via x, i.e., according to $\frac{d\widehat{q}_1}{dw_1} = \frac{\partial D_1}{\partial x} \cdot \frac{d\widehat{x}}{dw_1}$. Applying the implicit function theorem to the retailer's first-order condition to get $\frac{d\widehat{x}}{dw_1}$, yields

$$\frac{d\widehat{q}_1}{dw_1} = -\frac{\partial D_1}{\partial x} \cdot \frac{\frac{\partial^2 \pi_R}{\partial x \partial w_1}}{\frac{\partial^2 \pi_R}{\partial x^2}} = \frac{\left(\frac{\partial D_1}{\partial x}\right)^2}{\frac{\partial^2 \pi_R}{\partial x^2}} < 0,$$

so that the retailer's demand for M's good is strictly decreasing in the linear wholesale price w.

As we have derived equilibrium profits as functions of p_1 and w_1, namely,

$$\widehat{\pi}_M(p_1, w_1) := \widehat{q}_1(w_1 - c_1)$$

and

$$\widehat{\pi}_R(p_1, w_1) := \pi_R(p_1, \widehat{x}, w_1)$$

we can now turn to step 1b.

Step 1b: The manufacturer's problem. The manufacturer's maximization problem is

$$\max_{w_1, p_1 \geq 0} \widehat{\pi}_M(p_1, w_1) \quad \text{subject to} \quad \widehat{\pi}_R(p_1, w_1) \geq \pi_R^0. \tag{9}$$

We assume $\widehat{\pi}_M(p_1, w_1)$ to be quasi-concave. An interior solution, in which the retailer's constraint is not fulfilled as an equality, can be ruled out because even if there is such a candidate outcome, then for a higher or lower value of p_1 the demand for good 1 must increase strictly in one of the directions because of Assumption 2. Thus, the manufacturer will end up on the retailer's isoprofit curve, where $\widehat{\pi}_R = \pi_R^0$ holds.

As the retailer's participation constraint must hold as an equality in the optimal solution, $d\widehat{\pi}_R = d\pi_R^0 = 0$ must also hold, as π_R^0 is a constant. Hence,

$$\frac{\partial \widehat{\pi}_R}{\partial w_1} dw_1 + \frac{\partial \widehat{\pi}_R}{\partial p_1} dp_1 = 0,$$

which yields the slope of the retailer's isoprofit curve (fixed at π_R^0):

$$\left.\frac{dp_1}{dw_1}\right|_{\widehat{\pi}_R = \pi_R^0} = -\left.\frac{\frac{\partial \widehat{\pi}_R}{\partial w_1}}{\frac{\partial \widehat{\pi}_R}{\partial p_1}}\right|_{\widehat{\pi}_R = \pi_R^0}. \tag{10}$$

Note that $\frac{\partial \widehat{\pi}_R}{\partial w_1} = \left.\frac{\partial \pi_R}{\partial w_1}\right|_{x=\widehat{x}} + \frac{\partial \pi_R}{\partial x}\frac{d\widehat{x}}{dw_1} = -\widehat{q}_1 < 0$, as $\frac{\partial \pi_R}{\partial x} = 0$, so that the retailer's profit decreases in the wholesale price w_1. Thus, the sign of the slope of R's isoprofit curve (fixed at π_R^0) (see (10)) is given by the sign of $\frac{\partial \widehat{\pi}_R}{\partial p_1} = \left.\frac{\partial \pi_R}{\partial p_1}\right|_{x=\widehat{x}}$ (again, using the retailer's first-order condition). By Assumption 1, R's profit is strictly concave in p_1, so that the sign of $\frac{\partial \widehat{\pi}_R}{\partial p_1}$ can be positive or negative. If p_1 is set close to w_1 so as to reduce the double-markup inefficiency, then an increase in p_1 should increase the retailer's profit. If, to the contrary, p_1 is far above w_1 so as to induce the retailer to better internalize the vertical externality caused by his other decision variable, then a further increase of p_1 should affect the retailer's profit negatively.

In the optimal constrained solution, the total differential of the manufacturer's profit fulfills

$$\frac{\partial \widehat{\pi}_M}{\partial w_1} dw_1 + \frac{\partial \widehat{\pi}_M}{\partial p_1} dp_1 = 0,$$

subject to $\widehat{\pi}_R = \pi_R^0$, which gives the slope of the manufacturer's isoprofit curve (fixed at π_R^0):

$$\left.\frac{dp_1}{dw_1}\right|_{\widehat{\pi}_R = \pi_R^0} = -\left.\frac{\frac{\partial \widehat{\pi}_M}{\partial w_1}}{\frac{\partial \widehat{\pi}_M}{\partial p_1}}\right|_{\widehat{\pi}_R = \pi_R^0}. \tag{11}$$

In the constrained solution a marginal wholesale price change affects the manufacturer's profit positively; i.e., $\frac{\partial \widehat{\pi}_M}{\partial w_1} > 0$. Suppose otherwise: if it is negative, then the manufacturer will lower w_1, which is then always feasible as this would increase the profit of both the retailer and the manufacturer; if it is zero, then $\frac{\partial \widehat{\pi}_M}{\partial p_1} \neq 0$ (otherwise, we would be in an interior solution), so that the manufacturer has a strict incentive to lower w_1 (which does not affect the manufacturer's profit much and is feasible because this increases the retailer profit) and at the same time to change the price p_1 in the direction of $sign\left(\frac{d\widehat{q}_1}{dp_1}\right)$. For the manufacturer, the latter effect is of first-order, and the former is of second-order. The manufacturer, therefore, can clearly increase her profit while keeping the retailer indifferent. Thus, we have

$$\left.\frac{\partial\widehat{\pi}_M}{\partial w_1}\right|_{\widehat{\pi}_R=\pi_R^0} > 0.$$

It follows that the sign of (11) depends on the sign of $\frac{\partial\widehat{\pi}_M}{\partial p_1}$. The marginal effect of a retail price change p_1 on the manufacturer's profit is given by

$$\frac{\partial\widehat{\pi}_M}{\partial p_1} = \frac{d\widehat{q}_1}{dp_1}(w_1 - c_1),$$

so that

$$sign\left(\left.\frac{\partial\widehat{\pi}_M}{\partial p_1}\right|_{\widehat{\pi}_R=\pi_R^0}\right) = sign\left(\frac{d\widehat{q}_1}{dp_1}\right), \tag{12}$$

because $w_1 > c_1$ is obviously a property of the optimal contract. Notably, (12) says that in the optimal solution, the sign of the manufacturer's isoprofit curve (11) is determined by the sign of the slope of the induced demand of good 1. Thus, if the ID-case applies, the manufacturer's profit is increasing in the retail price p_1, because a higher price induces a higher value of x, so that the demand for M's product increases. Moreover, M's isoprofit curve must be downward sloping in this case, because M's profit always increases in the wholesale price w_1. If to the contrary, the DD-case holds, then M's profit is decreasing in the retail price because of the standard double-markup problem, in which case M's isoprofit curve is upward sloping.

We will use the relations (10), (11), and (12) in the next step to derive the optimal RPM contract.

Step 2: From price-fixing RPM to min RPM and max RPM. In the constrained solution of the price-fixing RPM contract, the right-hand side of (10) must be equal to the right-hand side of (11), which gives the optimality condition

$$-\left.\frac{\frac{\partial\widehat{\pi}_R}{\partial w_1}}{\frac{\partial\widehat{\pi}_R}{\partial p_1}}\right|_{\widehat{\pi}_R=\pi_R^0} = -\left.\frac{\frac{\partial\widehat{\pi}_M}{\partial w_1}}{\frac{\partial\widehat{\pi}_M}{\partial p_1}}\right|_{\widehat{\pi}_R=\pi_R^0}. \tag{13}$$

In the optimal solution, the manufacturer realizes the highest possible isoprofit curve, which must be tangent to the retailer's isoprofit curve (fixed at π_R^0). We can directly infer the optimal

RPM clause from condition (13). Suppose the ID-case with $\frac{d\widehat{q}_1}{dp_1} > 0$ applies, then $\frac{\partial \widehat{\pi}_M}{\partial p_1}\Big|_{\widehat{\pi}_R = \pi_R^0} > 0$ must hold because of (12), so that the manufacturer's isoprofit curve is downward sloping. Then the retailer's isoprofit curve must also slope downwards because of the optimality condition (13), which—in turn—requires that the retailer's profit must decrease when p_1 increases; i.e., $\frac{\partial \widehat{\pi}_R}{\partial p_1}\Big|_{\widehat{\pi}_R = \pi_R^0} < 0$. As we assumed that $\frac{\partial^2 \pi_R}{\partial p_1^2} < 0$ holds (Assumption 1), it follows that the retailer only wants to lower the retail price p_1 below the price-fixing solution because his profit only increases in this direction. Thus, a min RPM suffices to implement the optimal price-fixing contract whenever the ID-case applies.[10] Intuitively, if the ID-case holds, then the manufacturer wants to raise the price p_1 to a very high level to induce a favorable adjustment of the retailer's other decision variable (which is either a service increase or a price increase of the other product) such that the retailer only wants to reduce the retail price (and with that, the level of the other decision variable).

If, to the contrary, the DD-case with $\frac{d\widehat{q}_1}{dp_1} < 0$ holds, then the manufacturer's profit decreases in p_1 (i.e., $\frac{\partial \widehat{\pi}_M}{\partial p_1}\Big|_{\widehat{\pi}_R = \pi_R^0} < 0$), while the retailer's profit now must increase in p_1 (i.e., $\frac{\partial \widehat{\pi}_R}{\partial p_1}\Big|_{\widehat{\pi}_R = \pi_R^0} > 0$); again because of the optimality condition (13). Thus, a max RPM suffices to sustain the optimal solution whenever the DD-case applies because the retailer now only wants to raise p_1. The following proposition summarizes these results.

Proposition 1 (Main RPM result). *The manufacturer's profit-maximizing price-fixing contract (w_1, p_1) satisfies $\widehat{\pi}_R(p_1, w_1) = \pi_R^0$ and the optimality condition (13). Depending on whether or not the ID-case applies according to Assumption 2, either a min RPM or a max RPM suffices to sustain the profit-maximizing price-fixing solution:*

i) If the ID-case holds, then $\frac{\partial \widehat{\pi}_R}{\partial p_1} = \frac{\partial \pi_R}{\partial p_1}\Big|_{x=\widehat{x}} < 0$ follows from (13); i.e., a min RPM is used to sustain the manufacturer's profit-maximizing solution.

ii) If the DD-case holds, then $\frac{\partial \widehat{\pi}_R}{\partial p_1} = \frac{\partial \pi_R}{\partial p_1}\Big|_{x=\widehat{x}} > 0$ follows from (13); i.e., a max RPM is used to sustain the manufacturer's profit-maximizing solution.

Proposition 1 mirrors the fact that the manufacturer faces a tradeoff between reducing the

[10] Note that the assumption that the retailer's profit is strictly concave in p_1 (under the second-order condition) ensures that the retailer's isoprofit curve is always connected (i.e., there cannot be two unconnected isoprofit curves for the same profit level).

double-markup problem and incentivizing the retailer to better internalize the vertical externality that comes from his other decision variable. To overcome the double-markup problem it suffices to use a max RPM to reduce the retailer's margin, because the retailer only wants to increase the retail price in the constrained solution. To give the retailer incentives to internalize the vertical externality that comes from his other decision variable x, it suffices to use a min RPM in the constrained solution, because now the retailer only wants to reduce the retail price (and the value of his other decision variable) to increase his profit.

Proposition 1 also shows that the resolution of the tradeoff depends on the *induced* demand of the retailer, which results from the retailer's optimal choice of his additional decision variable. An RPM is always used to increase the sales quantity of the manufacturer's product. It then follows that a min (max) RPM is chosen whenever the induced demand for the manufacturer's good is upward (downward) sloping in its own retail price.

The slope of the induced demand (4) critically depends in the indirect demand effect caused by the optimal adjustment of the retailer's other decision variable x in response to a change of p_1. If this indirect demand effect is larger than the direct demand effect of a change of p_1, then the vertical externality associated with the retailer's additional decision variable x is relatively more important than the double-mark-up problem, so that the induced demand slopes upward. It is, therefore, intuitive that a min RPM (max RPM) is optimal whenever the vertical externality associated with the retailer's additional decision variable is relatively more (less) important for the manufacturer's sales quantity than the double-markup problem. In the min RPM case, M sets a relatively large retail margin, $p_1 - w_1$, which induces the retailer to set such a high value of x so that the retailer only wants to lower the price (and the level of x) to increase his profit. In the max RPM case, the vertical externality with the retailer's other decision variable is much less important so that M focuses on limiting the retail margin, in which case the retailer only wants to raise the retail price.

Our RPM result critically depends on our assumption that the manufacturer can only extract profits from the retailer through a linear wholesale price. Using an RPM increases the manufacturer's ability to extract rents from the retailer and induces the retailer to better internalize the vertical externality of his other decision variable. Because of the tradeoff between overcoming the double-marginalization problem and incentivizing the retailer, an RPM typically does not maximize the joint surplus of M and R. However, if we assume an efficient contract—for instance, a two-part tariff—, then M can easily implement the joint surplus maximizing solution by setting the wholesale price equal to her marginal production costs and by setting the fixed payment to the maximal level that keeps the retailer indifferent between accepting and rejecting the offer. In this case, the retailer implements the joint surplus maximizing solution as in the case of vertical integration. An RPM clause is not needed when a two-part tariff can be used.

Finally, we show how our result relates to Romano's (1994) seminal analysis of RPM in a successive monopoly with double-sided moral hazard.

Relation to Romano (1994). Romano assumes that there is a double moral hazard problem; i.e., both M and R are making non-contractible decisions after M has made a two-part tariff offer that R has accepted. Here, the final demand is, therefore, a function of three variables $D(p, x, y)$, where x stands for R's and y for M's non-contractible choice variable, respectively. Thus, our model for the "service case" is simply Romano's model, but with the vertical externality relating to M's non-price choice variable being removed and a contractual inefficiency (linear wholesale price) added in place. Note now that linear tariffs and upstream moral hazard both tend to cause double marginalization. According to Romano's Proposition 3, a min RPM (max RPM) is optimal if $E > 0$ ($E < 0$), where

$$E := \theta(-\epsilon_p + \epsilon_x \eta_x) + (1 - \theta)\epsilon_y \eta_y. \tag{14}$$

Here, θ represents the ratio of (equilibrium) wholesale-cost to retail price-cost margins, with $\theta := (w - c)/(p - c) \in [0, 1]$, for the case when RPM is banned.[11] Clearly, $\theta > 0$, whenever there is some degree of double marginalization. Note next that the expression inside the first parenthesis in (14), $-\epsilon_p + \epsilon_x \eta_x$, becomes identical to (4) whenever the effect of the upstream choice variable y disappears (except that in Romano it is multiplied by p/q to present it as an elasticity). Thus, abstracting from y (i.e., moral hazard on M'side) and assuming double marginalization ($\theta > 0$), the incentives for min RPM and max RPM are given by the sign of the

[11]The other variables represent elasticities in total values; in particular, ϵ_p is the price elasticity of demand, ϵ_x is the service elasticity of demand, and η_x denotes the price elasticity of equilibrium service. We can neglect the remaining two elasticities ϵ_y and η_y, which relate to M's noncontractible quality choice.

first term in parenthesis in (14), $-\epsilon_p + \epsilon_x \eta_x$, which is equivalent to the sign of the right-hand side of equation (4). Interestingly, Romano also points out how the sign of his expression (14) relates to the optimally used RPM type (p. 461): *"Then, ignoring for the moment the effect on the third externality (which can vanish), the prescribed nature of RPM depends on the net effect on the two externalities as measured by $\theta(-\epsilon_p + \epsilon_x \eta_x)$. The final price would be forced down (up) if the term in parentheses were negative (positive)."*

In conclusion, while our main RPM result can be inferred from Romano's analysis, our optimality condition (13) stays to be instructive, because—together with R's participation constraint—it directly pins down the equilibrium contract M will choose when an RPM clause is feasible. In the examples presented below, we directly derive the equilibrium outcome by applying (13).

4 Relation to Cost Pass-Through Analysis

The previous section shows that the sign of the slope of the induced demand uniquely determines whether a manufacturer wants to use a min or max RPM. This sign, however, is not observable itself, especially as wholesale prices tend to be unobservable for the empiricist. We show here that it is precisely a regulatory environment that bans RPM, which allows for a relatively easy way to infer this sign. We obtain this by examining the retailer's cost pass-through behavior.

For that purpose, assume that only the wholesale price w_1 is exogenously fixed. Accordingly, the retailer solves the "unconstrained" problem $\max_{p_1, x \geq 0} \pi_R$, so that the first-order conditions $\frac{\partial \pi_R}{\partial p_1} = 0$ and $\frac{\partial \pi_R}{\partial x} = 0$ hold. Applying the implicit function theorem, one gets the optimal price effect of a marginal change of the exogenous wholesale price, i.e., $\frac{dp_1}{dw_1}$. We speak of a positive cost pass-through when "$\frac{dp_1}{dw_1} > 0$" holds, and of a negative cost pass-through when "$\frac{dp_1}{dw_1} < 0$" holds.

It is easily checked that the ID-case with $\frac{d\widehat{q}_1}{dp_1} > 0$ holds if and only if the cost pass-through is negative, i.e., $\frac{dp_1}{dw_1} < 0$ holds. If, to the contrary, the cost pass-through is positive, $\frac{dp_1}{dw_1} > 0$, then the DD-case must apply.

Proposition 2 (Cost pass-through result). *The slope of the induced demand can be inferred* *from the sign of the cost pass-through for good 1 under an RPM ban. The ID-case with* $\frac{d\widehat{q}_1}{dp_1} > 0$ *holds according to (4) if and only if the cost pass-through is negative, i.e.,* $\frac{dp_1}{dw_1} < 0$ *holds; in this case* $\frac{dx}{dw_1} < 0$ *must also hold. The DD-case with* $\frac{d\widehat{q}_1}{dp_1} < 0$ *holds according to (4) if and only if the cost pass-through is positive; i.e.,* $\frac{dp_1}{dw_1} > 0$ *holds; in this case,* $\frac{dx}{dw_1} < 0$ *or* $\frac{dx}{dw_1} > 0$ *are both possible.*

Proof. See Appendix.

Proposition 2 implies that the sign of the slope of induced demand can be inferred from a cost pass-through analysis under a regime where the manufacturer can only set a linear wholesale price. Because of Proposition 1, we then also know the type of RPM the manufacturer will use if the RPM ban is lifted. If the cost pass-through is negative (positive), then a min (max) RPM is optimal for the manufacturer under a liberalized regime.

While there is a growing theoretical and empirical literature on cost pass-through in retailing, the particular problem we are interested in—namely, the product-specific cost pass-through under multiproduct retailing—remains largely unaddressed. One reason is that the literature often assumes a single product environment (see, e.g., Weyl and Fabinger, 2013), or assumes a demand system that rules out a negative cost pass-through (e.g., Moorthy, 2005, Assumption 1 on the demand system).

The empirical study by Besanko et al. (2005), however, documents negative pass-through rates. More precisely, the authors have retail and wholesale prices of a supermarket chain in Chicago with a market share of approximately 20%. The authors analyze pass-through rates for 11 product categories (like bathroom tissue, beer, canned tuna, laundry detergents, ...). They show that negative estimated own-brand elasticities are not uncommon (they occur — except for beer — for all considered product categories; see Table 3, p. 131, and Figure 2, p. 132). In terms of significance, the authors state that "5.6% of our estimates are negative and significant" (p.130).

Relation to Edgeworth Taxation Paradox and Bibliographical Note. Another aspect of Proposition 2 is that the induced demand $\widehat{q}_1$ is upward sloping if and only if the (unconstrained) retailer's optimal response to an exogenous wholesale price increase w_1 is to lower *both* the price of good 1, p_1, and x (the price of the other good or the selling services). For the multiproduct case, the negative cost pass-through scenario is reminiscent of Edgeworth's (1925) taxation paradox, which says that a multiproduct monopolist may want to reduce all prices when the marginal cost (a per-unit tax) of one good increases.[12] According to the logic of the Edgeworth taxation paradox an increase of w_1 induces the retailer to drive consumers from good 1 to good 2. For this to happen, the retailer reduces the price of good 2, which reduces demand for good 1, which—in turn—makes a price reduction for good 1 optimal. Thus, the demand for good 1 is reduced even though the retailer has reduced the price of good 1 in the course of his response to the marginal cost increase of good 1. According to Proposition 2, we then also know that the induced demand for good 1 is increasing in its retail price under a min RPM contract.

Salinger (1991) is the first paper that deals with the vertical integration of a manufacturer and a multiproduct retailer under conditions of a negative cost pass-through; i.e., when the Edgeworth taxation paradox applies. Such a merger would provide an incentive to steer customers to the internal good without double marginalization by increasing the price of the substitute good; a phenomenon honored by Luco and Marshall (2020) by coining the name "Edgeworth-Salinger effect" for it. However, in light of Edgeworth's negative cost pass-through result, not only the substitute good's price, but all prices could increase because of vertical integration, which—according to Salinger (1991)—challenges conventional wisdom that vertical integration is welfare-improving because it prevents double marginalization.

Our analysis shows that the logic of the Edgeworth taxation paradox can also emerge when the retailer decides on the selling services (and, more generally, could always apply in a setting where the downstream firm determines the product's quality as in Spence, 1975). In this case, a higher wholesale price reduces the retailer's selling services which leads to a reduced demand for good 1, which—in turn—makes a price reduction optimal. Again, in the course of the retailer's optimal response to the wholesale price increase, the demand for the manufacturer's good is reduced even though the retailer has lowered the retail price. From Proposition 2, we also know that the induced demand is upward-sloping, so the manufacturer will impose a min RPM when

[12]See Hotelling (1932) for an instructive discussion and the fact that it cannot be ruled out by standard assumptions; see also Garver (1933) for a critical view and Hotelling's (1933) response, Coase (1946) for a graphical exposition, and Vickrey (1960) for an extension to perfect competition. Recently, Armstrong and Vickers (2022) have derived additional results on this phenomenon.

feasible.

5 Two Linear Examples

We illustrate our previous results for the two cases using two examples with linear demands. The first example refers to the multiproduct case when demands are linear in prices. It shows that consumers are always hurt by a min RPM when the demand system is such that the manufacturer's induced demand is increasing in its own retail price; or equivalently when the cost pass-through would be negative under an RPM ban. The second example refers to the service case when the demand for the manufacturer's good is linear in its own price and the retailer's service. It shows that consumers always benefit from an optimally imposed RPM clause, while the social welfare assessment is less clear-cut.

5.1 The Multiproduct Case

Suppose the inverse demands are given by

$$p_1 = \max\{a_1 - b_1 q_1 - d_1 q_2, 0\} \text{ and} \tag{15}$$

$$p_2 = \max\{a_2 - d_2 q_1 - b_2 q_2, 0\}, \tag{16}$$

for two goods $1, 2$ with parameters $b_i > d_i > 0$ for $i = 1, 2$. Instead of x, here we use the variable p_2. We assume in the following a parameter range, which ensures the existence and uniqueness of the (interior) equilibrium solutions under both contracting regimes, where the manufacturer can only set a wholesale price and where the manufacturer can, in addition, set a min or max RPM. Inverting the (inverse) demand system (15)-(16) to get the demand functions $q_1 = D_1(p_1, p_2)$ and $q_2 = D_2(p_1, p_2)$, it is straightforward to get the derivatives

$$\frac{\partial D_1}{\partial p_1} = -\frac{b_2}{b_1 b_2 - d_1 d_2}, \frac{\partial D_1}{\partial p_2} = \frac{d_1}{b_1 b_2 - d_1 d_2}, \frac{\partial^2 \pi_R}{\partial p_2 \partial p_1} = \frac{d_1 + d_2}{b_1 b_2 - d_1 d_2}, \text{ and } \frac{\partial^2 \pi_R}{\partial p_2^2} = -\frac{2b_1}{b_1 b_2 - d_1 d_2}.$$

Substituting into (6) gives

$$\frac{d\widehat{q}_1}{dp_1} = \frac{d_1^2 + d_2 d_1 - 2b_1 b_2}{2b_1 (b_1 b_2 - d_1 d_2)}.$$

Note that $2b_1(b_1b_2 - d_1d_2) > 0$. Thus, $\frac{d\widehat{q_1}}{dp_1} > 0$ holds if[13]

$$d_1^2 + d_2d_1 - 2b_1b_2 > 0, \tag{17}$$

which requires $d_1 > b_2$ to hold as well. If and only if condition (17) holds, the manufacturer wants to set a min RPM according to Proposition 1.[14]

Proposition 3. *If the demand functions are linear, the manufacturer sets a min RPM when the ID-case (17) holds, that is, $d_1^2 + d_2d_1 - 2b_1b_2 > 0$. When the DD-case holds, that is, $d_1^2 + d_2d_1 - 2b_1b_2 < 0$, she sets a max RPM.*

We can compare the "price-fixing regime" according to Proposition 1 with the successive monopoly outcome in the absence of an RPM (which we refer to as the "linear wholesale pricing regime"); that is, when the manufacturer can only set a linear wholesale price w_1, while the retailer sets both retail prices p_1 and p_2 to maximize his profits (1). We then get the following result.

Proposition 4. *The manufacturer charges the same wholesale price under the price-fixing and the linear wholesale pricing regimes.*

i) If the ID-case holds, the manufacturer sets a min RPM such that all market prices increase above the prices that prevail under the linear wholesale pricing regime. In this case, consumers are worse off.

ii) If the DD-case holds, the manufacturer sets a max RPM such that the price of good 1 decreases while the price for good 2 can in- or decrease when compared with the linear wholesale pricing regime.

The proof of this proposition is straightforward along the following lines. Solving the game for the case that the manufacturer sets only a linear wholesale price, and for the case that the

[13]Notably, the integrability condition $\partial p_1/\partial q_2 = \partial p_2/\partial q_1$ does not hold here, so these demand functions cannot be derived from a representative-agent model; but with non-linear demand curves, Edgeworth's paradox can also arise when the integrability conditions holds (see Hotelling, 1932). This is closely related to the discussion on the symmetry of the Slutsky matrix: while the Slutsky symmetry is predicted by the classical model, it is rejected by a large body of empirical literature (see, for instance, the thorough discussion of Slutsky symmetry in Gabaix, 2014).

[14]Condition (17) is compatible with Assumption 1.

manufacturer can also fix the retail price, we get the same wholesale price[15]

$$w_1 = \frac{1}{2}(a_1 + c_1) - \frac{d_1 + d_2}{4b_2}(a_2 - w_2). \tag{18}$$

It then follows that all final good prices increase if condition (17) holds by use of a min RPM relative to the case that the manufacturer can only set a linear wholesale price. This also implies that with the use of a min RPM, consumers are clearly worse off (simply by revealed preferences).

We note that the condition (17) for a min RPM to arise under a linear demand system (15)-(16) is rather restrictive. It requires that the demand for the manufacturer's good is more sensitive (in absolute terms) to the other good's price than to her own price; that is, $d_1 > b_2$ must hold. Nevertheless, in the following example, we show that a min RPM can also arise when the retailer's additional decision variable relates only to promotional services for the manufacturer's good.

5.2 The Service Case

Here we use the primitives of a simple service free riding model (see, Motta, 2004, p. 316). Suppose R faces a consumer demand for M's product given by

$$q_1 := D_1(p_1, x) = 1 - p_1 + x, \tag{19}$$

where x stands for R's selling effort. The costs of the selling effort are given by $C(x) = \frac{t}{2}x^2$.[16] Accordingly, the retailer's profit function is given by $\pi_R = (1 - p_1 + x)(p_1 - w_1) - \frac{t}{2}x^2$. The second-order conditions of R's unconstrained problem $\max_{p_1,x} \pi_R$ require $t > 1/2$, which we assume in the following.

Inspection of R's induced demand under a price-fixing contract shows that the ID-case (DD-case) holds for $t < 1$ ($t > 1$). We thus know according to Proposition 1 that M's optimal contract implies a min RPM (max RPM) for $t < 1$ ($t > 1$).

[15] The manufacturer's profit-maximizing price-fixing contract (w_1, p_1) satisfies $\widehat{\pi}_R = \pi_R^0$ and the optimality condition (13), from which we get the wholesale price as stated in (18).

[16] Another specification—proposed by Mathewson and Winter (1984)—would be to assume that the retailer undertakes (informative) advertising efforts, which increase demand at a decreasing rate, while they come at linear costs. Our results stay qualitatively valid under that approach.

Applying the optimality condition (13) at $\widehat{\pi}_R(p_1, w_1) = 0$, we get the optimal price floor $p_1^{\min} = \frac{1+c_1-4t}{2(1-2t)}$ if M wants to implement a min RPM and the optimal price ceiling $p_1^{\max} = \frac{1+c_1}{2}$ if M wants to implement a max RPM. It is easily checked that the optimal wholesale price is always given by $w_1^* = \frac{1+c_1}{2}$, and that this is also the optimal wholesale price in the absence of an RPM clause.

The intuition behind these solutions is simple. A min RPM is used whenever retail services are relatively efficient (precisely: $t < 1$ holds). In this case, the manufacturer sets a retail price above the wholesale price (i.e., $p_1^{\min} - w_1^* > 0$) to induce the retailer to undertake selling services ($x > 0$ holds in the equilibrium with a min RPM). If a min RPM was not feasible, R would provide a lower level of services and charge a lower retail price.

A max RPM is optimal for M to set whenever retail services are relatively inefficient (precisely: $t > 1$ holds). This allows M to remove the retailer's margin, as a consequence of which service provision is also zero (i.e., in equilibrium, we have $p_1^{\max} = w_1^*$ and $x = 0$).

We next state the consumer and social welfare effects of min and max RPM relative to linear pricing (for which we use the indices min, max, and LW, resp.).

Proposition 5. *Consumer surplus, CS, is larger under a min and max RPM relative to the case where the manufacturer can only set a linear wholesale price; i.e., $CS^{\min} > CS^{LW}$ for all $t < 1$ and $CS^{\max} > CS^{LW}$ for all $t > 1$ (with equality holding at $t = 1$).*

Proposition 5 corresponds to the benign assessment of a min RPM as being desirable from a consumer perspective when it induces retail selling efforts. Interestingly, in our example, retailer services only occur under a min RPM and do not play any role under a max RPM. In the latter case, consumers benefit from a max RPM because the manufacturer avoids the double-markup inefficiency.

For social welfare, we get the following result.

Proposition 6. *The comparison of social welfare under a min and max RPM relative to the case where the manufacturer can only set a linear wholesale price is as follows:*

i) If $t > 1$ (i.e., a max RPM is used), then $SW^{\max} > SW^{LW}$ if $t > \frac{1}{10}\sqrt{21} + \frac{9}{10} \approx 1.36$ and $SW^{\max} < SW^{LW}$ if $t < \frac{1}{10}\sqrt{21} + \frac{9}{10} \approx 1.36$.

ii) If $t < 1$ (i.e., a min RPM is used), $SW^{\min} > SW^{LW}$ if $t < \frac{1}{14}\sqrt{3}\sqrt{7} + \frac{1}{2} \approx 0.83$ and

$SW^{\min} < SW^{LW}$ *if* $t > \frac{1}{14}\sqrt{3}\sqrt{7} + \frac{1}{2} \approx 0.83$.

Proposition 6 qualifies the unequivocally positive assessment of a min and a max RPM. Both a min and a max RPM can be used too often from a social welfare perspective, which is most likely when the service efficiency (as measured by t) is at an intermediary level close to $t = 1$.

6 Generalization

In this section, we generalize our model toward a setting where the retailer has not only one, but several additional decision variables at hand (that are other goods' prices and product-specific selling services). Basically, all our main results stay valid under a straightforward adaptation of Assumption 1 to such a generalized setting (see Assumption 1' presented in the Online Appendix). This holds, in particular, for the cost pass-through result (Proposition 2). We then examine an example where the retailer offers a second good and provides selling services for M's product 1. By that, we can combine within a single example the pro- and anticompetitive effects associated with a min RPM we have highlighted in the previous two examples.

Generalized set-up. Suppose the retailer sells $n \geq 1$ products indexed by $i = 1, ..., n$, where $i = 1$ is the product offered by M. Let $\overline{p} := (p_1, ..., p_n)$ be the price vector, which assigns to every product i a retail price p_i. In addition, the retailer offers product-specific services for products $i = 1, ..., m$, with $m \leq n$. Let $\overline{s} := (s_1, ..., s_m)$ be the service vector, which assigns to every product i the selling service of the retailer s_i.

Demand for good i is given by $q_i := D_i(\overline{p}, \overline{s})$. We assume that demand is downward sloping in its own price and increasing in the other goods' prices; i.e., $\frac{\partial D_i}{\partial p_i} < 0$ and $\frac{\partial D_i}{\partial p_i'} > 0$ hold for all $i = 1, ..., n$ and $i \neq i'$. Regarding the selling services, we suppose that demand of product i is increasing in its own service and decreasing in the other products' services; i.e., $\frac{\partial D_i}{\partial s_i} > 0$ and $\frac{\partial D_i}{\partial s_{i'}} < 0$ hold for all $i = 1, ..., m$ and $i \neq i'$.

The manufacturer M sets the wholesale price w_1, while all other wholesale prices $w_2, ..., w_n$ are exogenously given. Let $C(\overline{s})$ be the service cost function with $\frac{\partial C}{\partial s_i} > 0$ for all $i = 1, ..., m$. Thus, the retailer's profit is given by

$$\pi_R(\overline{p}, \overline{s}) := \sum_{i=1}^{n} [D_i(\overline{p}, \overline{s})(p_i - w_i)] - C(\overline{s}). \tag{20}$$

We assume that in the retailer's "unconstrained" profit-maximizing solution—that is, in the absence of a price-fixing RPM clause—all first-order conditions hold as equalities; i.e., $\frac{\partial \pi_R}{\partial p_i} = 0$ for all $i = 1, ..., n$, and $\frac{\partial \pi_R}{\partial s_i} = 0$ for all $i = 1, ..., m$, which is ensured by assuming that all second-order conditions are fulfilled (see Assumption 1' in the Online Appendix). By assumption, we rule out corner solutions so that the retailer's optimal decisions involve strictly positive quantities and strictly positive services for all considered products and services.[17]

It then follows that the retailer's first-order conditions for $p_2, ..., p_n$ and $s_1, ..., s_m$ also hold as equalities under a price-fixing RPM contract. By the implicit function theorem, we can then write the induced demand for product 1 as a function of p_1,

$$\widehat{q}_1 = D_1(p_1, \widehat{p}_2(p_1), ..., \widehat{p}_n(p_1), \widehat{s}_1(p_1), ..., \widehat{s}_m(p_1)), \tag{21}$$

where $\widehat{p}_i(p_1)$, for $i = 2, ..., n$, and $\widehat{s}_i(p_1)$, for $i = 1, ..., m$, are the implicit functions that follow from the retailer's first-order conditions. Totally differentiating (21) with respect to p_1 gives the slope of the induced demand

$$\frac{d\widehat{q}_1}{dp_1} = \frac{\partial D_1}{\partial p_1} + \sum_{i=2}^{n} \frac{\partial D_1}{\partial p_i} \cdot \frac{d\widehat{p}_i}{dp_1} + \sum_{i=1}^{m} \frac{\partial D_1}{\partial s_i} \cdot \frac{d\widehat{s}_i}{dp_1}, \tag{22}$$

so that a marginal change of the retail price of good 1 now leads to a composite of indirect price and service effects (given by the second and third term on the right-hand side of (22), respectively), which can countervail the negative direct effect (given by the first term on the right-hand side of (22)). For instance, for $n = m = 2$, a marginal price increase of good 1 increases demand for good 1 via three indirect channels: first, by raising the other good's price (for $\frac{d\widehat{p}_2}{dp_1} > 0$), second by raising services for good 1 (for $\frac{d\widehat{s}_1}{dp_1} > 0$), and third by reducing the services for the second good (for $\frac{d\widehat{s}_2}{dp_1} < 0$).

We posit that Assumption 2 on the monotonicity of the induced demand also holds in the general setup. It then follows that the entire analysis of the manufacturer's maximization problem and Proposition 1 (see Section 3) on the use of a min/max RPM also applies here.

[17]We regard this assumption as quite innocent. A corner solution could only occur if the considered set of products/services is suboptimally extended. As we are not concerned with the optimal composition of products (product variety) and services, we can therefore—from the start—define the set of considered products and services as the one of which positive levels are chosen in the optimum by the retailer.

Likewise, the relation between the RPM type (min or max) and the cost pass-through under an RPM ban (see Proposition 2) stays valid.

Proposition 7 (Cost pass-through result for n goods and m services). *Assume $n \geq 1$ and $m \geq 0$, with $n + m \geq 2$. The relation between the slope of the induced demand for good 1 under a price-fixing contract, $\frac{d\widehat{q}_1}{dp_1}$, and the cost pass-through, $\frac{dp_1}{dw_1}$, in the absence of an RPM contract is given by*

$$\frac{d\widehat{q}_1}{dp_1} = \frac{|A|}{|B|} \cdot \frac{dp_1}{dw_1}, \text{ with } \frac{|A|}{|B|} < 0,$$

where $|A|$ and $|B|$ are the determinants of the Hessian matrix associated with the retailer's unconstrained and RPM-constrained maximization problem, respectively.

Proof. See Online Appendix.

As the second-order conditions of the unconstrained retailer problem are fulfilled (by Assumption 1' in the Online Appendix) it must also hold that the determinants of the Hessian matrices A and B have opposite signs. By the relation stated in Proposition 7, it must then also hold that the slope of the induced demand with respect to its own price, $\frac{d\widehat{q}_1}{dp_1}$, must have the reverse sign of the cost pass-through of the own wholesale price, $\frac{dp_1}{dw_1}$, under an RPM ban.

With those results at hand, we can turn next to an example where the retailer offers a second good and a selling service for good 1; i.e., we can combine the anticompetitive effect of a min RPM vis-à-vis the second substitute product (see the multiproduct example above) and the procompetitive effect with regard to the retailer's services (see the service example above) within a single example.

Example with two goods and one service. Suppose a representative consumer model with a (quasi-linear) quadratic utility function that allows for vertical product differentiation (Häckner, 2000) given by

$$U(q_1, q_2, s_1) := q_1(1 + s_1) + q_2 - \frac{1}{2}\left(q_1^2 + q_2^2\right) - \frac{1}{2}q_1 q_2.$$

Thus, goods 1 and 2 are symmetrically differentiated in the absence of selling services ($s_1 = 0$), while product 1 becomes vertically differentiated when the retailer exerts services ($s_1 > 0$). The demand functions are then given by

$$q_1 := D_1(p_1, p_2, s_1) := \frac{2(1 + 2s_1 - 2p_1 + p_2)}{3} \text{ and} \tag{23}$$

$$q_2 := D_2(p_1, p_2, s_1) := \frac{2(1 - s_1 - 2p_2 + p_1)}{3}. \tag{24}$$

Let the service cost function for good 1 be $C(s_1) = \frac{t}{2}s_1^2$. Moreover, let $c_1 = 0$ and $w_2 = 0$, so that neither of the goods has a cost advantage. In the following, we assume $t > 1$, which ensures that the second-order conditions (see Assumption 1' in the Online Appendix) hold and that the solution is interior.

If the retailer accepts M's offer, then the retailer's profit function is given by

$$\pi_R(p_1, p_2, s_1) = \sum_{i=1}^{2} D_i(p_1, p_2, s_1)(p_i - w_i) - C(s_1), \tag{25}$$

while the retailer's outside option can be calculated to be $\pi_R^0 = \frac{1}{4}$. Solving the retailer's problem under a price-fixing RPM contract, we get the induced demand for good 1

$$\widehat{q}_1 = \frac{5t - 2 - (8 + t)w_1 + 2(4 - 3t)p_1}{6t - 1}, \text{ with} \tag{26}$$

$$\frac{d\widehat{q}_1}{dp_1} = \frac{2(4 - 3t)}{6t - 1},$$

so that $\frac{d\widehat{q}_1}{dp_1} \gtreqless 0 \Leftrightarrow t \lesseqgtr \frac{4}{3}$; that is, M finds a min RPM (max RPM) optimal whenever $t < \frac{4}{3}$ ($t > \frac{4}{3}$) holds.

Solving the manufacturer's maximization problem $\max_{w_1, p_1 \geq 0} \widehat{\pi}_M(p_1, w_1)$ subject to the retailer's participation constraint, $\widehat{\pi}_R(p_1, w_1) = \frac{1}{4}$, by applying the optimality condition (13), we get the optimal min and max RPM contracts. A comparison with the solution of M's optimal wholesale price under an RPM ban shows that the optimal wholesale price for good 1 is in all cases the same, namely, $w_1 = \frac{1}{4}$, while prices, quantities and service differ.

Proposition 8. *If the manufacturer uses a min RPM (max RPM) when strictly optimal, then all prices and the service level are higher (lower) than under an RPM ban. Sales quantity of product 1 is always larger under either type of RPM contract than in the presence of an RPM ban. Sales quantity of product 2 is lower (higher) under a min RPM (max RPM) than under an RPM ban.*

Proof. See Online Appendix.

Proposition 8 reiterates that an RPM contract is always used to increase the sales quantity of product 1. In the case of a min RPM, this is achieved by a relatively high retail price for good 1, which induces the retailer to intensify his service efforts for product 1 on the one hand and to raise the second good's price to shift demand to product 1 on the other hand. In the case of a max RPM—which is used when services are relatively costly—the RPM clause is used to overcome the double-markup problem, which reduces all prices and the level of service efforts. The overall effect on output is positive for both products as selling services are relatively unimportant in this case.

The following result summarizes the consumer surplus and social welfare effects of RPM relative to a market regime where RPM is banned.

Proposition 9. *If the manufacturer uses a min RPM, then there exist unique critical values* $t_{CS}^{\min}$ *and* $t_{SW}^{\min}$*, with* $1 < t_{SW}^{\min} < t_{CS}^{\min} < \frac{4}{3}$*, such that consumer surplus (social welfare) is larger under a min RPM than under an RPM ban if* $t < t_{CS}^{\min}$ *(*$t < t_{SW}^{\min}$*) holds, while the opposite is true in the reverse case. If the manufacturer uses a max RPM, consumer surplus always increases relative to a regime where RPM is banned. Finally, there exists a unique critical value* $t_{SW}^{\max}$*, with* $t_{SW}^{\max} > \frac{4}{3}$*, such that social welfare is lower (higher) if the manufacturer uses a max RPM than under an RPM ban if* $t < t_{SW}^{\max}$ *(*$t > t_{SW}^{\max}$*).*

Proof. See Online Appendix.

Proposition 9 qualifies the unambiguously positive assessment of a min RPM from a consumer perspective when the retailer offers services but does not offer a second substitute good (see the example for the service case above). With a second good at hand, a min RPM not only incentivizes the retailer to perform more selling services but also raises the price of the substitute good. While the first effect is generally procompetitive and in the consumers' interests, the second effect unambiguously harms consumers. According to Proposition 9, the first (procompetitive) effect is relatively less pronounced the less efficient the service technology is (i.e., t is relatively high, with $t > t_{CS}^{\min}$), so that the overall effect of a min RPM on consumer surplus can be negative. If, however, the service technology is sufficiently efficient ($t < t_{CS}^{\min}$), then a min RPM increases consumer surplus. Finally, taking account of producer surplus tends to make an RPM ban even more desirable as the parameter range where a positive assessment of the social welfare effect of a min RPM holds shrinks even further (i.e., $t_{SW}^{\min} < t_{CS}^{\min}$).

With regard to a max RPM—which applies to the DD-case—our results call competition authorities' generally lenient attitude towards max RPM in question (seen as being procompetitive by nature). While consumers still profit from an optimally used max RPM in our example, social welfare can decrease if the service technology is sufficiently efficient (i.e., $t < t_{SW}^{\max}$).

7 Conclusion

We derive a new rationale for why a manufacturer sets a min RPM, namely, a non-contractible choice variable of the retailer, which can be the price of a substitute good or the retailer's sales effort. According to our analysis, it is essential to see what the exact reason for a min RPM is, as this determines whether a min RPM benefits or harms consumers. In the service case, it benefits consumers as it increases the delivered service level. But in the multiproduct case, a min RPM can be detrimental to consumer welfare even though it increases sales volume for the respective product. This is in contrast to what the literature stated, whereby a min RPM should be beneficial as long as it does not lower sales (see, e.g., Posner, 1981, or Elzinga and Mills, 2008, that sum up on p. 9: "If putting an RPM policy in place boosts total sales noticeably, this strongly suggests that consumers, on net, have benefited."). Similarly, Klein (2009, p. 449) argues that an RPM allows for a Pareto-improving allocation by incentivizing the retailer to "provide the level of manufacturer-specific promotional efforts that maximizes manufacturer profitability. This incentive incompatibility between the manufacturer and its retailers creates a profitable opportunity for manufacturers to design distribution arrangements whereby retailers are compensated for supplying increased manufacturer-specific promotional efforts."

In light of our analysis, Grimes' (2010) critique of such a "profitability/output test" for a procompetitive assessment of an RPM is a valid one; at least, when several brands compete for shelf space and promotional efforts at the retailer's premises. In those instances, the negative output effect of a min RPM on the sales quantities of rival products has to be taken into account.

In the course of our analysis, we have also uncovered a relation between the cost pass-through analysis (which is closely related to the economics of the Edgeworth taxation paradox in the multiproduct retailing case) and a min RPM, which has so far gone unnoticed. Augmenting the archetypal successive monopoly model with a substitute product the retailer has at hand, we could show that a min RPM is optimal for the manufacturer whenever the cost pass-through under an RPM ban is negative (which is possible not only in the multiproduct case used by Edgeworth to show his result, but also in the service case). Only then, the retailer's induced demand for the manufacturer's good is increasing in its retail price from which the incentive to impose a min RPM follows. This nexus is not just a theoretical curiosity but also points out an avenue to infer the likely effects of lifting the ban on RPM, which is currently in place in the EU and many other jurisdictions; namely, by conducting cost pass-through studies which take full account of retailers' non-contractible decisions (be it the pricing of substitute products or the allocation of selling services). Our analysis should be helpful for such an undertaking because our main results extend to cases where the retailer sells many substitute goods and decides on product-specific selling services.

Finally, taking a dynamic perspective, the availability of a min RPM should enhance manufacturers' incentives to invest in branded goods when the brands are sold via multiproduct retailers to final consumers. With a min RPM the manufacturer can realize higher sales volumes and profits so that incentives to develop new branded goods and to invest in "brand image" are strengthened.

Appendix

Proof of Proposition 2. Consider the retailer problem when only the wholesale price w_1 is exogenous. The retailer solves $\max_{p_1,x\geq 0}\pi_R$, where π_R is given either by (1) or by (2). Given Assumption 1, the optimal values of p_1 and x follow from the retailer's first-order conditions, $\frac{\partial \pi_R}{\partial p_1}=0$ and $\frac{\partial \pi_R}{\partial x}=0$. Totally differentiating the first-order conditions with respect to w_1 and solving for $\frac{dp_1}{dw_1}$, one gets the condition for a negative cost pass-through (incidentally, this is the original formulation of the condition for the Edgeworth taxation paradox; see Bailey, 1954, Selten, 1970, and Salinger, 1991):

$$\frac{dp_1}{dw_1} = \frac{\frac{\partial D_1}{\partial p_1}\frac{\partial^2 \pi_R}{\partial x^2} - \frac{\partial D_1}{\partial x} \cdot \frac{\partial^2 \pi_R}{\partial x \partial p_1}}{\frac{\partial^2 \pi_R}{\partial p_1^2}\frac{\partial^2 \pi_R}{\partial x^2} - \left(\frac{\partial^2 \pi_R}{\partial p_1 \partial x}\right)\left(\frac{\partial^2 \pi_R}{\partial x \partial p_1}\right)} < 0. \tag{27}$$

The denominator is positive (second-order condition, see Assumption 1), so that $\frac{dp_1}{dw_1} < 0$ holds if and only if the numerator is negative. We can re-write $\frac{d\widehat{q}_1}{dp_1}$ (see (6)) as

$$\frac{d\widehat{q}_1}{dp_1} = \frac{1}{\frac{\partial^2 \pi_R}{\partial x^2}}\left(\frac{\partial D_1}{\partial p_1} \cdot \frac{\partial^2 \pi_R}{\partial x^2} - \frac{\partial D_1}{\partial x} \cdot \frac{\partial^2 \pi_R}{\partial x \partial p_1}\right), \tag{28}$$

so that the sign of $\frac{d\widehat{q}_1}{dp_1}$ is given by the reverse sign of the term in brackets on the right-hand side of (28), which is the same term as the term in the numerator of (27) (note that $\frac{\partial^2 \pi_R}{\partial x^2} < 0$; Assumption 1). It is then straightforward to see that $sign\left(\frac{d\widehat{q}_1}{dp_1}\right) = -sign\left(\frac{dp_1}{dw_1}\right)$. □

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Online Appendix

Proof of Proposition 7. Define the vector of the retailer's decision variables by $\overline{y} = (y_1, ..., y_K)$, with $\overline{y} := (\overline{p}, \overline{s})$, so that $K = n + m$. Let $k = 1, ..., K$ be the index for the K decision variables of the retailer, where $k = 1, ..., n$ indicates the goods' prices p_i and $k = n+1, ..., n+m$ indicates the retailer's services s_i for products $i = 1, ..., m$. Note that $y_1 \equiv p_1$.

We assume that in the retailer's unconstrained profit maximizing solution—in the absence of a price-fixing RPM clause—all first-order conditions hold as equalities; i.e., $\frac{\partial \pi_R}{\partial y_k} = 0$ for all $k = 1, ..., K$, which is ensured by the following assumption.

Assumption 1' (Second-order conditions). *The second-order conditions of the retailer's unconstrained maximization problem* $\max_{y_1,...,y_K \geq 0} \pi_R(\overline{p}, \overline{s})$ *hold; i.e.,*

$$\pi_{11} < 0, \quad \begin{vmatrix} \pi_{11} & \pi_{12} \\ \pi_{21} & \pi_{22} \end{vmatrix} > 0, \quad \begin{vmatrix} \pi_{11} & \pi_{12} & \pi_{13} \\ \pi_{21} & \pi_{22} & \pi_{23} \\ \pi_{31} & \pi_{32} & \pi_{33} \end{vmatrix} < 0, ..., \ (-1)^k \cdot \begin{vmatrix} \pi_{11} & \pi_{12} & ... & \pi_{1k} \\ \pi_{21} & \pi_{22} & ... & \pi_{2k} \\ ... & ... & ... & ... \\ \pi_{k1} & \pi_{k2} & ... & \pi_{kk} \end{vmatrix} > 0,$$

for all $k \leq K$*, where* $\pi_{kk} := \frac{\partial^2 \pi_R}{\partial y_k^2}$ *and* $\pi_{kk'} := \frac{\partial^2 \pi_R}{\partial y_k \partial y_{k'}}$ *for* $k \neq k'$.

From now on, the proof proceeds in four steps. In step 1, we derive the cost pass-through, $\frac{dp_1}{dw_1}$, under an RPM ban, where M only controls the wholesale price w_1. In step 2, we derive the optimal adjustments of the retailer's decision variables, $y_2, ..., y_K$, under a price-fixing RPM contract in response to a marginal change of good 1's retail price p_1; i.e., $\frac{d\widehat{y}_k}{dp_1}$, for all $k = 2, ..., K$. In step 3, we obtain the slope of the induced demand and in step 4 we show that the relation stated in Proposition 7 holds for any number of products $n \geq 1$ and any number of services $m \leq n$.

Step 1. Comparative statics of the unconstrained maximization problem w.r.t. w_1. R solves $\max_{y_1,...y_K \geq 0} \pi(\overline{y})$. All first-order conditions $\frac{\partial \pi_R}{\partial y_k} = 0$, for $k = 1, ..., K$, hold as equalities. By the implicit function theorem, the optimal values of the retailer's decision variables $y_1, ..., y_K$ can be written as implicit functions of w_1; i.e., $\widehat{y}_1(w_1), ..., \widehat{y}_K(w_1)$. Thus, totally differentiating the system of first-order conditions, $\frac{\partial \pi_R}{\partial y_k} = 0$, for $k = 1, ..., K$, with respect to w_1 we get

$$\underbrace{\begin{pmatrix} \pi_{11} & \pi_{12} & ... & \pi_{1K} \\ \pi_{21} & \pi_{22} & ... & \pi_{2K} \\ ... & ... & ... & ... \\ \pi_{K1} & \pi_{K2} & ... & \pi_{KK} \end{pmatrix}}_{=:A} \begin{pmatrix} \frac{d\widehat{p}_1}{dw_1} \\ \frac{d\widehat{y}_2}{dw_1} \\ ... \\ \frac{d\widehat{y}_K}{dw_1} \end{pmatrix} = \begin{pmatrix} -\frac{\partial^2 \pi}{\partial p_1 \partial w_1} \\ -\frac{\partial^2 \pi}{\partial y_2 \partial w_1} \\ ... \\ -\frac{\partial^2 \pi}{\partial y_K \partial w_1} \end{pmatrix} = \begin{pmatrix} \frac{\partial D_1}{\partial p_1} \\ \frac{\partial D_1}{\partial y_2} \\ ... \\ \frac{\partial D_1}{\partial y_K} \end{pmatrix}, \tag{29}$$

where the last equality follows from noticing that $\frac{\partial \pi_R}{\partial y_k} = 0$ implies $\frac{\partial^2 \pi_R}{\partial y_k \partial w_1} = -\frac{\partial D_1}{\partial y_k}$ for all $k = 1, ..., K$. Define the $K \times K$ Hessian matrix on the left-hand side of (29) as A. Next define by A_1 the matrix that follows from replacing the first column of matrix A by the vector of the right-hand side of (29); i.e.,

$$A_1 := \begin{pmatrix} \frac{\partial D_1}{\partial y_1} & \pi_{12} & ... & \pi_{1K} \\ \frac{\partial D_1}{\partial y_2} & \pi_{22} & ... & \pi_{2K} \\ ... & ... & ... & ... \\ \frac{\partial D_1}{\partial y_K} & \pi_{K2} & ... & \pi_{KK} \end{pmatrix}. \tag{30}$$

By Cramer's rule, we then get the own-price effect of a marginal change of good 1's wholesale price (i.e., the cost pass-through of good 1):

$$\frac{dp_1}{dw_1} = \frac{|A_1|}{|A|}.$$

Step 2. Comparative statics of the constrained maximization problem w.r.t p_1. Under a price-fixing RPM contract M sets the retail price of good 1, p_1. If the retailer accepts this contract, then R solves $\max_{y_2,...y_K \geq 0} \pi(\overline{y})$. All first-order conditions $\frac{\partial \pi_R}{\partial y_k} = 0$, for $k = 2, ..., K$, hold as equalities. Again, by the implicit function theorem, the optimal values of the retailer's decisions $y_2, ..., y_K$ can be written as implicit functions of p_1; i.e., $\widehat{y}_2(p_1), ..., \widehat{y}_K(p_1)$. Totally differentiating the $K - 1$ first-order conditions with respect to the retail price of good 1, p_1, gives

$$\underbrace{\begin{pmatrix} \pi_{22} & ... & \pi_{2K} \\ ... & ... & ... \\ \pi_{K2} & ... & \pi_{KK} \end{pmatrix}}_{=:B} \begin{pmatrix} \frac{d\widehat{y}_2}{dp_1} \\ ... \\ \frac{d\widehat{y}_K}{dp_1} \end{pmatrix} = \begin{pmatrix} -\pi_{21} \\ ... \\ -\pi_{K1} \end{pmatrix}. \tag{31}$$

We define the $(K-1) \times (K-1)$ matrix on the left-hand side of (31) by B, which is the Hessian matrix associated with the retailer's constrained maximization problem. Note that matrix B is the submatrix of A which is formed by deleting the first row and the first column of A. Applying Cramer's rule to (31), we get

$$\frac{d\widehat{y}_k}{dp_1} = \frac{|B_k|}{|B|}, \text{ for } k \geq 2,$$

where B_k is obtained from matrix B by substituting the k-th column (now with $k = 2, ..., K$) by the column vector of the right-hand side of (31); for instance,

$$B_2 = \begin{pmatrix} -\pi_{21} & \pi_{23} & ... & \pi_{2K} \\ -\pi_{31} & \pi_{33} & ... & \pi_{3K} \\ ... & ... & ... & ... \\ -\pi_{K1} & \pi_{K3} & ... & \pi_{KK} \end{pmatrix}.$$

Step 3. The slope of the induced demand under a price-fixing RPM contract. By the implicit function theorem, we can write the induced demand for product 1 as a function of p_1; namely,

$$\widehat{q}_1 = D_1(p_1, \widehat{y}_2(p_1), \widehat{y}_3(p_1), ..., \widehat{y}_K(p_1)). \tag{32}$$

Totally differentiating (32) with respect to p_1 gives

$$\frac{d\widehat{q}_1}{dp_1} = \frac{\partial D_1}{\partial p_1} + \sum_{k=2}^{K} \frac{\partial D_1}{\partial y_k} \cdot \frac{d\widehat{y}_k}{dp_1}. \tag{33}$$

Substituting $\frac{d\widehat{y}_k}{dp_1} = \frac{|B_k|}{|B|}$, for $k = 2, ..., K$ into (33), we get

$$\frac{d\widehat{q}_1}{dp_1} = \frac{1}{|B|} \left[|B| \cdot \frac{\partial D_1}{\partial p_1} + \sum_{k=2}^{K} \frac{\partial D_1}{\partial y_k} \cdot |B_k| \right]. \tag{34}$$

Step 4. We complete the proof by showing that the term in rectangular brackets on the right-hand side of (34) is equal to $|A_1|$, which implies that $\frac{d\widehat{q}_1}{dp_1} = \frac{|A|}{|B|} \frac{dp_1}{dw_1}$ holds as stated in the proposition, while the reverse relation of the signs of $\frac{d\widehat{q}_1}{dp_1}$ and $\frac{dp_1}{dw_1}$ follows from the assumed second order conditions (see Assumption 1' above).

We, therefore, have to prove the following claim to complete the proof.

Claim. $|B| \cdot \frac{\partial D_1}{\partial p_1} + \sum_{k=2}^{K} \frac{\partial D_1}{\partial y_k} \cdot |B_k| = |A_1|$ *is true for all* $K \geq 2$.

Proof. Using the Laplace expansion along the first column vector of matrix A_1 (see (30)), the determinant of A_1 is given by

$$|A_1| = \sum_{k=1}^{K} \frac{\partial D_1}{\partial y_k}(-1)^{k+1} |M_{k1}|, \tag{35}$$

where $|M_{k1}|$ is the minor of the $(k,1)$-element of $|A_1|$. We, therefore, have to show that

$$|B| \cdot \frac{\partial D_1}{\partial p_1} + \sum_{k=2}^{K} \frac{\partial D_1}{\partial y_k} \cdot |B_k| = \sum_{k=1}^{K} \frac{\partial D_1}{\partial y_k}(-1)^{k+1} |M_{k1}| \tag{36}$$

is true. There are K terms on the left-hand side and on the right-hand side of (36), which we compare one by one according their order of appearance.

Case $k = 1$ (first terms). For $k = 1$, we get $M_{11} = B$, so that the first term on the right-hand side, $\frac{\partial D_1}{\partial p_1}(-1)^{1+1} |M_{11}|$, is equal to the first term on the left-hand side, $|B| \cdot \frac{\partial D_1}{\partial p_1}$, of (36).

Case $k = 2$ (second terms). The second term on the left-hand side of (36) is given by $\frac{\partial D_1}{\partial y_2} \cdot |B_2|$, with

$$B_2 = \begin{pmatrix} -\pi_{21} & \pi_{23} & \ldots & \pi_{2K} \\ -\pi_{31} & \pi_{33} & \ldots & \pi_{3K} \\ \ldots & \ldots & \ldots & \ldots \\ -\pi_{K1} & \pi_{K3} & \ldots & \pi_{KK} \end{pmatrix}.$$

The second term on the right-hand side of (36) is given by $\frac{\partial D_1}{\partial y_2}(-1)^{2+1} |M_{21}|$, with

$$M_{21} = \begin{pmatrix} \pi_{12} & \pi_{13} & \ldots & \pi_{1K} \\ \pi_{32} & \pi_{33} & \ldots & \pi_{2K} \\ \ldots & \ldots & \ldots & \ldots \\ \pi_{K2} & \pi_{K3} & \ldots & \pi_{KK} \end{pmatrix}.$$

Multiplying the first column vector of B_2 by -1 and transposing the resulting matrix gives M_{21}, so that $|B_2| = -|M_{21}|$.[18] It follows that $\frac{\partial D_1}{\partial y_2} \cdot |B_2| = -\frac{\partial D_1}{\partial y_2} |M_{21}|$.

[18] The determinants of a matrix and its transpose are equal. Multiplying a column vector by -1 or interchanging any two columns changes only the sign of the determinant. Note also that by Young's theorem, $\pi_{kk'} = \pi_{k'k}$ for $k' \neq k$.

Case $k = 3$. Proceeding as before, we get

$$B_3 = \begin{pmatrix} \pi_{22} & -\pi_{21} & \pi_{24} & ... & \pi_{2K} \\ \pi_{32} & -\pi_{31} & \pi_{34} & ... & \pi_{3K} \\ ... & ... & ... & ... & ... \\ \pi_{K2} & -\pi_{K1} & \pi_{K4} & ... & \pi_{KK} \end{pmatrix}$$

and

$$M_{31} = \begin{pmatrix} \pi_{12} & \pi_{13} & ... & \pi_{1K} \\ \pi_{22} & \pi_{23} & ... & \pi_{2K} \\ \pi_{42} & \pi_{43} & ... & \pi_{4K} \\ ... & ... & ... & ... \\ \pi_{K2} & \pi_{K3} & ... & \pi_{KK} \end{pmatrix}.$$

We get from B_3 to M_{31} via the following operations: First, we multiply the second column (i.e., the $k = 3$-th column) of B_3 by -1 which only changes the sign of the determinant. Second, we interchange columns $k = 2$ and $k = 3$ (i.e., the first and the second columns of B_3), which only changes the sign of the determinant. Third, we transpose the resulting matrix to get M_{31}. Thus, $|B_3| = |M_{31}|$, so that the third terms on either side of (36) are equal: i.e., $\frac{\partial D_1}{\partial y_3} \cdot |B_3| = \frac{\partial D_1}{\partial y_3} |M_{31}|$.

Case $k = 4$. We get from B_4 to M_{41} by the same three operations as for the case $k = 3$ (i.e., by multiplying the $k = 4$-th column by -1, interchanging columns, and transposing the resulting matrix). However, while we needed for $k = 3$ only to interchange the first and second column, we need one more such operation for $k = 4$, because when moving the third column vector ($k = 4$) into the first place ($k = 2$), we also have to interchange the new second column with the first column to get them into an ascending order in terms of k. As we now need two such permutations to get M_{41}, the sign of the determinant is not affected by these operations together. Thus, only multiplying the k-th column vector by -1, affects the sign of the determinant and we get $|B_4| = -|M_{41}|$. We then also get $\frac{\partial D_1}{\partial y_4} \cdot |B_4| = -\frac{\partial D_1}{\partial y_4} |M_{41}|$, so the fourth terms on either side of the equation are equal.

Cases $k = 6, 8, ...$ (all remaining even terms). For all remaining even terms $k = 6, 8, ...$, we need an even number of permutations to move the k-th column into the first place while reordering the remaining columns into an ascending order in k. Thus, the sign of the determinant of B_k is not affected by all those permutations together. The only operation which changes to sign of the determinant is the multiplication of the k-th column vector by -1. It then follows that $|B_k| = -|M_{k1}|$ for all $k = 6, 8, ...$ It then also follows that $\frac{\partial D_1}{\partial y_k} \cdot |B_k| = -\frac{\partial D_1}{\partial y_k} |M_{k1}|$ for all $k = 6, 8, ...$

Cases $k = 5, 7, ...$ (all remaining uneven terms). For all uneven cases $k = 5, 7, ...$, we need an uneven number of permutations to move the k-th column into the first while reordering the remaining columns in an ascending order. Thus all those permutations together only change the sign of the determinant of B_k. As we also have to multiply the k-th column vector by -1 to get from B_k to M_{k1}, we get that $|B_k| = |M_{k1}|$ holds in all those cases. It is then immediate that $\frac{\partial D_1}{\partial y_k} \cdot |B_k| = \frac{\partial D_1}{\partial y_k} |M_{k1}|$ for all $k = 5, 7, ...$ □

Proof of Propositions 8 and 9 (third example: two goods, one service). We here present the calculations of the third example (two goods, one service) from which the results stated in Propositions 8 and 9 follow. It is easily checked that the second-order conditions (see Assumption 1' above) hold for $t > 2/3$. To ensure a strictly positive output level of good 2 for all $w_1 \geq 0$ under a linear wholesale price regime, we have to assume $t > 1$, which implies that all solutions are interior.

We first solve for the equilibrium under a linear wholesale price when RPM is banned altogether. We then solve for the RPM solution of the game and finally derive the orderings and results stated in Propositions 8 and 9.

Linear wholesale price regime. Solving the retailer's problem, $\max_{p_1,p_2,s_1 \geq 0} \pi_R$, where π_R follows from inserting the demands (23)-(24), the service cost function $C(s_1) = \frac{t}{2} s_1^2$, and $w_2 = 0$ into (25), gives the retailer's optimal decisions $p_1(w_1)$, $p_2(w_1)$, and $s_1(w_1)$ as functions of w_1. With those solutions, we can calculate the derived demand for good 1, which is $\widehat{q}_1(w_1) = \frac{t(1-2w_1)}{3t-2}$. Solving then M's problem, $\max_{w_1 \geq 0} \widehat{q}_1(w_1) w_1$, yields the equilibrium wholesale price $w_1^{LW} = \frac{1}{4}$, where the superscript LW stands for the linear wholesale price regime. Plugging this value into $p_1(w_1)$, $p_2(w_1)$, and $s_1(w_1)$, we get the equilibrium values of the retailer's decision variables

$$p_1^{LW} = \frac{15t - 8}{8(3t - 2)},\ p_2^{LW} = \frac{1}{2},\ \text{and}\ s_1^{LW} = \frac{1}{2\,(3t - 2)},$$

and the equilibrium output levels

$$q_1^{LW} = \frac{t}{2(3t-2)} \text{ and } q_2^{LW} = \frac{5t-4}{4(3t-2)}.$$

RPM Case. If the retailer accepts M's contract offer—which consists of a wholesale price and a price-fixing RPM—, then her profit π_R follows again from inserting the demands (23)-(24), the service cost function $C(s_1) = \frac{t}{2}s_1^2$, and $w_2 = 0$ into (25). If the retailer rejects M's offer, then $s_1 = q_1 = 0$, so that the inverse demand for product 2 is given by $p_2^0 = 1 - q_2$. In this case, the retailer solves $\max_{q_2 \geq 0}(1-q_2)q_2$, which implies an outside option profit of $\pi_R^0 = \frac{1}{4}$.

If R accepts M's contract offer, then R solves $\max_{p_2, s_1 \geq 0} \pi_R$, which yields R's optimal decisions as functions of p_1 and w_1; i.e.,

$$\widehat{p}_2 = \frac{3t(1+2p_1-w_1) - 4(p_1-w_1)}{2(6t-1)} \text{ and } \widehat{s}_1 = \frac{6p_1 - 7w_1 - 1}{6t-1}. \tag{37}$$

Substituting (37) into (23), gives the induced demand of product 1 as stated in (26). Using all those results, we obtain the retailer's reduced profit as a function of p_1 and w_1:

$$\widehat{\pi}_R = \frac{t - 4p_1 + 4w_1 + 12tp_1 - 10tw_1 + 8p_1^2 + 8w_1^2 - 16p_1w_1 - 12tp_1^2 + tw_1^2 + 12tp_1w_1}{2(6t-1)}.$$

In the first stage of the game, the manufacturer solves $\max_{w_1, p_1 \geq 0} \widehat{\pi}_M$ subject to $\widehat{\pi}_R(p_1, w_1) \geq \pi_R^0 = \frac{1}{4}$, where $\widehat{\pi}_M = \widehat{q}_1 w_1$. The profit maximizing values of w_1 and p_1 fulfill the retailer's participation constraint (holding as an equality) and the optimality condition (13). To apply the optimality condition, we have to calculate the partial derivatives of R's and M's profit functions $\widehat{\pi}_R$ and $\widehat{\pi}_M$, with respect to w_1 and p_1, respectively. Solving the two equations for w_1 and p_1, we get the equilibrium values

$$p_1^{\min} = \frac{15t - 8 + \sqrt{2}\sqrt{t\,(6t-1)}}{8(3t-2)} \text{ and } w_1^{\min} = \frac{1}{4}.$$

for the min RPM solution and the equilibrium values

$$p_1^{\max} = \frac{15t - 8 - \sqrt{2}\sqrt{t\,(6t-1)}}{8(3t-2)} \text{ and } w_1^{\max} = \frac{1}{4}.$$

for the max RPM solution. Plugging these values into $\widehat{p}_2$ and $\widehat{s}_1$, we get the equilibrium values for the min RPM outcome

$$p_2^{\min} = \frac{4(6t-1) + \sqrt{2}\sqrt{t\,(6t-1)}}{8(6t-1)} \text{ and } s_1^{\min} = \frac{2(6t-1) + 3\sqrt{2}\sqrt{t\,(6t-1)}}{4(18t^2 - 15t + 2)},$$

and the equilibrium values for the max RPM outcome

$$p_2^{\max} = \frac{4(6t-1) - \sqrt{2}\sqrt{t\,(6t-1)}}{8(6t-1)} \text{ and } s_1^{\max} = \frac{2(6t-1) - 3\sqrt{2}\sqrt{t\,(6t-1)}}{4(18t^2-15t+2)}.$$

Substituting those values into the demands (23)-(24), we get for the min RPM outcome

$$q_1^{\min} = \frac{2t(6t-1) + \sqrt{2}\sqrt{t\,(6t-1)}(4-3t)}{4(18t^2-15t+2)} \text{ and}$$
$$q_2^{\min} = \frac{4 + 30t^2 - 29t - \sqrt{2}\sqrt{t\,(6t-1)}}{4(18t^2-15t+2)},$$

and for the max RPM outcome

$$q_1^{\max} = \frac{2t(6t-1) - \sqrt{2}\sqrt{t\,(6t-1)}(4-3t)}{4(18t^2-15t+2)} \text{ and}$$
$$q_2^{\max} = \frac{4 + 30t^2 - 29t + \sqrt{2}\sqrt{t\,(6t-1)}}{4(18t^2-15t+2)}.$$

Given those equilibrium values, it is straightforward to check that the inequalities

$$p_1^{\min} > p_1^{LW} > p_1^{\max},\ p_2^{\min} > p_2^{LW} > p_2^{\max},\ s_1^{\min} > s_1^{LW} > s_1^{\max},$$
$$q_1^{\min}, q_1^{\max} > q_1^{LW}, \text{ and } q_2^{\max} > q_2^{LW} > q_2^{\min}$$

hold—given that a min RPM (max RPM) is used when (strictly) optimal; i.e., when $t < \frac{4}{3}$ ($t > \frac{4}{3}$) holds. This proves Proposition 8.

Finally, we have to compare consumer surplus, with $CS := U(q_1, q_2, s_1) - \sum_{i=1}^{2} p_i q_i$, and social welfare, defined by $SW := CS + \pi_M + \pi_R$, for the three cases LW, min RPM, and max RPM, again assuming that the RPM clause is used optimally. We obtain the following values for consumer surplus

$$CS^{LW} = \frac{39t^2 - 48t + 16}{32\,(3t-2)^2},$$
$$CS^{\min} = \frac{170t - 369t^2 + 252t^3 - 16 + \left(36t - 8 - 27t^2\right)\left(\sqrt{2}\sqrt{t\,(6t-1)}\right)}{32\,(3t-2)^2\,(6t-1)},$$
$$CS^{\max} = \frac{170t - 369t^2 + 252t^3 - 16 - \left(36t - 8 - 27t^2\right)\left(\sqrt{2}\sqrt{t\,(6t-1)}\right)}{32\,(3t-2)^2\,(6t-1)},$$

and for social welfare

$$SW^{LW} = \frac{3\left(43t^2 - 52t + 16\right)}{32\left(3t-2\right)^2},$$

$$SW^{\min} = \frac{466t - 1077t^2 + 756t^3 - 48 + \left(72t - 45t^2 - 24\right)\left(\sqrt{2}\sqrt{t\left(6t-1\right)}\right)}{32\left(3t-2\right)^2\left(6t-1\right)}, \text{ and}$$

$$SW^{\max} = \frac{466t - 1077t^2 + 756t^3 - 48 - \left(72t - 45t^2 - 24\right)\left(\sqrt{2}\sqrt{t\left(6t-1\right)}\right)}{32\left(3t-2\right)^2\left(6t-1\right)}.$$

The comparison of consumer surplus under a min and max RPM relative to an RPM ban can be reduced to the equation

$$-4212t^5 + 11\,637t^4 - 10962t^3 + 4092t^2 - 622t + 64 = 0,$$

which has two complex and three real zeros $t_1 \approx 0.50575$, $t_2 \approx 1.0219$, and $t_3 \approx 1.0757$. It is then easily checked that $CS^{\min} > CS^{LW}$ holds for $t < t_3 =: t_{CS}^{\min}$ and that $CS^{\max} > CS^{LW}$ for all $t > \frac{4}{3}$ (i.e., when the max RPM is used optimally).

Proceeding likewise, the comparison of social welfare under a min and max RPM relative to an RPM ban can be reduced to the equation

$$23\,976t^5 - 82\,242t^4 + 101\,736t^3 - 55\,632t^2 + 13\,340t - 1152 = 0,$$

which has two complex and three real zeros $t_1 \approx 0.59431$, $t_2 \approx 1.014$, and $t_3 \approx 1.3341$. It is then easily checked that $SW^{\min} > CS^{LW}$ holds for $t < t_2 =: t_{SW}^{\min}$ and that $SW^{\max} < CS^{LW}$ holds for $t < t_3 =: t_{SW}^{\max}$, while the reverse orderings hold otherwise. This proves Proposition 9. □

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